Measurement of Quantum Fluctuations in Geometry

Craig J. Hogan

University of Washington, Seattle, WA 98195-1580, USA

A particular form for the quantum indeterminacy of relative spacetime position of events is derived from the limits of measurement possible with Planck wavelength radiation. The indeterminacy predicts fluctuations from a classically defined geometry in the form of "holographic noise" whose spatial character, absolute normalization, and spectrum are predicted with no parameters. The noise has a distinctive transverse spatial shear signature, and a flat power spectral density given by the Planck time. An interferometer signal displays noise due to the uncertainty of relative positions of reflection events. The noise corresponds to an accumulation of phase offset with time that mimics a random walk of those optical elements that change the orientation of a wavefront. It only appears in measurements that compare transverse positions, and does not appear at all in purely radial position measurements. A lower bound on holographic noise follows from a covariant upper bound on gravitational entropy. The predicted holographic noise spectrum is estimated to be comparable to measured noise in the currently operating interferometer GEO600. Because of its transverse character, holographic noise is reduced relative to gravitational wave effects in other interferometer designs, such as LIGO, where beam power is much less in the beamsplitter than in the arms.

I. INTRODUCTION

There have been to date no experiments revealing quantum behavior of spacetime. On the other hand it is possible that quantum geometrical effects— new behavior reflecting a quantum indeterminacy of geometry not present in the classical model of spacetime defined as a smooth manifold with definite paths between pointlike events— may be directly detectable in the form of spacetime fluctuations, particularly as a new source of noise in interferometers. Predicted effects, such as noise spectra in various instrumental setups, have been estimated in the context of a variety of theories of quantum gravity and Lorentz symmetry violation [1, 2, 3, 4, 5, 6, 7]. In this paper, a holographic hypothesis for quantum geometry is formulated that is more specific than previous phenomemological estimates of quantum-gravitational noise derived from those more general theoretical approaches. As shown here, this specificity leads to sharp predictions for the transverse spatial character, spectrum and absolute normalization of the noise, and parameter-free predictions for statistical properties of signals in real interferometers.

The motivation for the particular new form of quantum spacetime indeterminacy considered here originates in ideas for reconciling gravity and quantum mechanics built on the idea of holography[8, 9, 10, 11, 12, 13]. A holographic quantum geometry of spacetime has two spatial dimensions instead of three, and the apparent third dimension emerges, by a hologram-like encoding, along a null projection of a 2D sheet. Although holographic noise is not derived here from a fundamental theory, its observable properties are fixed by simple and general arguments of an essentially geometrical character, based on indeterminacy of measurements made using Planck wavelength radiation, with an absolute scale ultimately normalized by covariant bounds on geometrical entropy[14, 15, 16, 17]. Holographic quantum geometry implies a surprisingly large quantum indeterminacy in the relative transverse positions of events. These appear in the 3D world as apparent fluctuations in the metric with a distinctive, purely transverse spatial shear character not apparent in predictions of earlier phenomenological analyses [2, 3, 4, 5, 6], and a flat, frequency-independent spectrum, with a power spectral density of equivalent metric shear fluctuations given simply by the Planck time t_P.

This holographic noise appears to be observable in the signals of some operating interferometers. Although some experiments have been mounted expressly to search for quantum gravitational effects (e.g., [18]), the most sensitive interferometers are those built to detect gravitational waves[19]. These interferometers have now attained an important threshold of sensitivity: the measured spectral density of noise in the recent science runs of LIGO[20, 21] is less than $h = 10^{-22} \text{Hz}^{-1/2}$ over a broad band, from about 70 Hz to about 300 Hz. This number should be compared with the square root of the Planck time $t_P \equiv l_P/c \equiv \sqrt{\hbar G_N/c^5} = 5 \times 10^{-44}$ Hz^{-1}, where $\hbar$ denotes Planck's constant, G_N denotes Newton's constant, and c denotes the speed of light. Systems with metric strain noise below $h \approx \sqrt{t_P} = 2.3 \times 10^{-22} \text{Hz}^{-1/2}$ in principle have the capability of ruling out or possibly studying in detail effects arising at the Planck scale. The detectability of such effects is also critically dependent on details of interferometer design: the holographic noise discussed here appears only in comparisons of transverse positions, as occurs for example in reflection off a beamsplitter. For this reason, the most promising currently operating experiment for detecting the effect is not LIGO, but GEO600 [22]. The estimate below of the signal in equivalent gravitational wave spectral density predicts that the current GEO600 apparatus should display measurable holographic noise, and will allow a test of this class of holographic theories.

II. QUANTUM GEOMETRY DEFINED BY PLANCK WAVELENGTH RADIATION

The basic physical effect underlying holographic indeterminacy, uncertainty and noise can be captured in a simple wave model. The essential argument is that the uncertainty of spacetime is determined by the intrinsic indeterminacy of defining intervals between events using Planck wavelength radiation. The only physically significant quantities defining events in a spatially extended spacetime are those that in principle can be mapped using locally measured signals in Planck-wavelength interferometers. Holographic indeterminacy arises as a consequence of wave/particle complementarity in a spacetime defined by such Planck wavelength waves: it is the limiting precision to which worldlines can be measured with Planck radiation.

Consider a metric where the positions of events and paths are defined using only waves longer than a cutoff λ. Paths connecting events are then subject to indeterminacy because of the limitation of defining the endpoints of a ray, or a path, corresponding to any wave. Using the Rayleigh criterion, a particle path corresponding to a wave propagating over a length L has one endpoint within an aperture of size D and the other within a diffraction spot of size $\lambda L/D$. The range of possible orientations consistent with those endpoints is minimized when $D = \lambda L/D$. The endpoints of the ray are uncertain by an amount $\Delta x = \sqrt{\lambda L}$, corresponding to an aperture with the same size as its own diffraction spot at distance L. The orientation of a ray of wavelength λ over a length L can at best be defined with a precision $\Delta \theta = \sqrt{\lambda/L}$. This criterion defines an unavoidable classical transverse indeterminacy of rays that are defined by waves. We conjecture that the transverse indeterminacy of Planck wavelength quantum paths corresponds to quantum indeterminacy of the metric itself.

This argument is classical, but as it is essentially geometrical, it has a wide application. The same indeterminacy generalizes to any theory limited to quantities that are measured locally by comparing phases in a Planck wavelength interferometer. It also applies to rays or paths in a virtual 3D world encoded in a 2D hologram. It seems likely to be a feature of 3+1D spacetime emerging as a dual of quantum theory on a 2+1D null surface. As shown below, less uncertain transverse position would imply a number of degrees of freedom, as measured by the number of distinguishable position eigenstates, in excess of covariant holographic entropy bounds.

Consider events corresponding to interactions of Planck radiation on a null surface at two different times, at normal coordinates z_1 and z_2 in a particular frame. The radial separation $z_1 - z_2$ is measured to Planck precision, defining the relative z position between the events on each surface. Consider an event on surface 1, such as a Planck particle reflection. The particle obeys the usual Heisenberg commutation relation between conjugate momentum and position observable operators along the transverse x-axis:

$$[\hat{x}(z_1), \hat{p}_x(z_1)] = -i\hbar. \tag{1}$$

The transverse momentum $p_x(z_1)$ of the particle on surface 1 is related to a transverse position displacement on surface 2 by the angular deflection,

$$p_x(z_1) l_P/\hbar = x(z_2)/(z_2 - z_1). \tag{2}$$

Combining equations (1) and (2) yields a commutation relation between transverse position operators for events on the two surfaces, fixed by an elapsed time in the z direction,

$$[\hat{x}(z_1), \hat{x}(z_2)] = -i l_P(z_2 - z_1), \tag{3}$$

where Planck's constant $\hbar$ has dropped out.

This formula specifies the complementarity of the transverse position observables at macroscopic separation along a null trajectory, and thereby expresses the uncertainty of the null trajectory itself; quantum indeterminacy prevents more precise specification of the relative transverse positions of events. An interferometer with a beamsplitter can fold the beam and compare the two surfaces and the relative positions of events, $x(z_1) - x(z_2)$, directly; but even a Planck interferometer cannot eliminate the indeterminacy.

In the usual way, the indeterminacy (Eq. 3) yields a Heisenberg uncertainty relation:

$$\Delta x(z_1) \Delta x(z_2) > l_P(z_2 - z_1)/2, \tag{4}$$

where $\Delta x(z_1), \Delta x(z_2)$ denote the standard deviations on surfaces 1 and 2 of the wavefunction of transverse position measurements. The standard deviation $\Delta x_\perp$ of the difference in relative transverse positions is then given by $\Delta x_\perp^2 = \Delta x^2(z_1) + \Delta x^2(z_2)$; it has a minimum value when $\Delta x(z_1) = \Delta x(z_2)$. This defines a "holographic uncertainty principle" for relative transverse positions at events of null spacetime separation and spatial separation L in a given frame:

$$\Delta x_\perp^2 > l_P L. \tag{5}$$

From this we also derive a minimum uncertainty in angular orientation of a null ray of length L along each transverse axis:

$$\Delta \theta_x > \sqrt{l_P/L}. \tag{6}$$

This geometrical wave model quantitatively reproduces the spatial character of holographic indeterminacy without explicitly introducing a holographic principle or even referring to gravity directly, aside from the introduction of the Planck scale as a fundamental wavelength. Although the quantum evolution of such a system is deterministic and unitary, in the usual quantum mechanical way, measurement choices determine which branch of a wavefunction an observer lies on and break the symmetry of the wavefunction[23]. The choice of orientation of a

mirror at a given event— which determines the direction in which an apparatus measures that event's spacelike position relative to future events— fixes an eigenstate and collapses the future metric into states compatible with that measurement. In orthogonal directions the state is a superposition so measurements of an orthogonal position are indeterminate.

This quantum behavior differs significantly from the classical background metric assumed in quantum field theory. Angular uncertainty increases with smaller L, so that a classical spatial direction is ill defined at the Planck scale and only becomes well defined after many Planck lengths of propagation. Angles become better defined at larger macroscopic separation— in this sense the world becomes "more classical" as it becomes on larger scales "more three dimensional." What is surprising is that transverse positions in terms of absolute length actually become less well defined at larger separations. Transverse positions of macroscopically separated events do not exist as separately observable quantities, but are complementary: knowledge of one position precludes accurate knowledge of the other. Macroscopic spacetime limited by Planck scale measurements exhibits quantum departures from classical Euclidean behavior on scales significantly larger than the Planck length.

Measurements made locally on a single, spacelike slice of a null surface can be made at Planck resolution and collapse the system into a corresponding eigenstate. Events on the null projection of the surface— in the third spatial dimension— are encoded in this system at the same time. But in a particular eigenstate those remote events are fuzzy; they are defined with substantially worse than Planck resolution. This notion of a transverse position uncertainty of events, described as wavefunctions of interferometer optical surfaces, substitutes here for full correlators of observables.

Although we have treated this effect as a quantum indeterminacy, the same phenomenology can also be described as a Brownian-like random shear of null surfaces: Null surfaces (and null particle paths) execute a Planck random walk in the direction transverse to their direction of propagation. The displacement in the transverse direction accumulates over a path like a diffusion process; it does not cancel or converge to a classical value, which in any case could not be defined or measured to better precision even with Planck wavelength radiation. (The angular direction of a path does however converge to a well defined classical value on large scales, which is why a 3+1D approximation works well.) Similar stochastic descriptions are familiar in quantum systems[24].

III. HOLOGRAPHIC INDETERMINACY OF SIGNALS IN AN IDEALIZED INTERFEROMETER

A. Quantum Indeterminacy of Reflection Event Positions

The interferometer model provides a concrete framework for discussing this particular hypothesis about the quantum uncertainty of spacetime, and predicting observable effects. The model is built on the idea of a machine where the optical elements share the same transverse position uncertainty as Planck quanta, an indeterminacy that reflects quantum indeterminacy of spacetime itself. Relative positions of events are only physically distinguished to the extent that they can be measured with the Planck interferometer. This notion of spacetime position implies substantially greater indeterminacy in the transverse positions of events than the definition of intervals using null trajectories to connect worldlines and local clocks to measure separation of events[25, 26], or the limits imposed locally by measurements with Planck radiation[27]. The underlying reason for the noise is indeterminacy of 3D states defined by data on a two-dimensional surface, a property also illustrated by the holographic character of states in a field theory with a UV cutoff[28, 29].

In practical terms, measurement of holographic noise requires a comparison of transverse positions over a macroscopic interval, using elements of an interferometer— a real one, not one using Planck radiation. Interferometer phase measures the relative positions of events defined by photon interactions with pieces of macroscopically separated optical elements. The effect of the holographic noise is to add indeterminacy to the measurements of the position differences in two orthogonal directions. The hypothesis is that a real interferometer cannot have less noise than one using Planck radiation.

Consider a quantum-limited Planck wavelength interferometer with a beamsplitter (Figure 1). A quantum enters the apparatus at the top, and eventually is detected on the surface S. Its path is split into beams travelling down and right, sent in those directions and then returned after travelling a distance L out and back. After recombining at the beamsplitter, its arrival is recorded on a detector surface S. Each interfering branch of the particle path involves one reflection event on the beamsplitter, and the transverse location of the particle is indeterminate until it is detected. Classically, the arrival time of the particle does not depend on its transverse position: any of the various paths from one of the distant directions leads to an arrival at the same time. An interferometer measures the variations in the path difference to the two distant reflections by an average signal strength on S; for a single particle the arrival time probability distribution is proportional to signal strength.

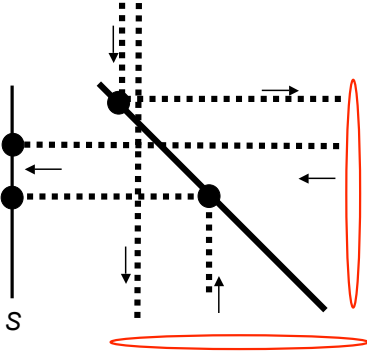The particle wavepacket is broadened into ellipses as

FIG. 1: Inteferometer showing two branches of a single self-interfering particle path with dotted lines. Reflection events on the beamsplitter, and detection events on the detector S, are shown by dots. After the return reflection, the incoming paths have an indeterminate transverse position indicated by the elliptical contours.

shown in

(1), but remains narrow in the propagation direction so the arrival at S happens "at the same time" (in the lab frame) whichever path is taken, even though the transverse arrival position is uncertain by $\Delta x = \sqrt{l_P L}$. Arrival events for various paths of a particle in a given wavefront have a spacelike separation and the invariant interval between them is distributed with a variance $\Delta x^2 \approx l_P L$. Classically, detection events on S inhabiting a null surface identified with a particular wavefront have spacelike separation from each other but all happen at the same time in the lab frame. The beamsplitter simply changes the propagation direction of the wavefront without changing the separations of the events normal to the surface.

In a holographic theory the reflection and detection events have a different relationship since in the holographic world everything lives on null surfaces, including the inclined reflecting surface of the beamsplitter. For a world description based on a null surface, the reflection events at different places on the 2D reflecting surface inclined with respect to the null surface do not behave with respect to each other like classical pointlike events. While the relative positions of events on one slice of the 2D null surface (corresponding to a wavefront at one time in the lab frame) have relative transverse locations defined to Planck precision, the transverse positions of other events on different slices of the same surface, projected into the third (virtual) dimension, are not pointlike but are indeterminate. Their 3D location is encoded nonlocally in a patch or wavepacket of a size that grows with the separation between slices. In 3D space the patches correspond to wavefunctions that encode relative transverse positions of events, such as those of the same wavefront reflecting off different parts of the beamsplitter, located on different slices of the null surface. (In holographic dual models this indeterminacy is described using using quantum correlators.)

Holographic uncertainty shares much in common with the fuzziness seen in real holograms. A simple description of holographic uncertainty is that transverse spacetime positions themselves share the same indeterminacy that would attach to Planck wavelength quanta. Thus the position of an optical surface is uncertain between outgoing and incoming reflections by $\Delta x = \sqrt{l_P L}$ where L is the path length between reflections. A transverse uncertainty of beamsplitter position relative to either arm gives the same effect on the final signal: measured arrival times of wavefronts separated by t have an uncertainty with standard deviation $\Delta t \approx \sqrt{t_P t}$.

B. Random Walk of Signal Phase

The position of any body, including an optical element such as the beamsplitter of an interferometer, is encoded holographically on a null surface. If we choose the surface to coincide with incoming quanta, the reflection events are in eigenstates of position in that direction. The quanta are then reflected in an orthogonal direction, placing the reflection event in an eigenstate of position in an orthogonal direction. Each effectively independent measurement of a quantum arriving on surface S resets the metric to a new eigenvalue of relative position between S and a beamsplitter element. Each independent reflection event adds an uncertain arrival time offset and these offsets accumulate in quadrature, instead of averaging to a fixed classical value as they would in a spacetime with a classically fixed beamsplitter position. The beamsplitter position as determined by a phase signal measuring a difference in position in the two directions appears to undergo a random walk with time.

When we compare the descriptions of the same world on two orthogonal null surfaces, the relative positions of events specified to lie on a single slice of one null surface are on different slices of the other. When a reflection occurs, a system in an eigenstate of one direction is measured (and "collapses") into an eigenstate in another direction. An uncertainty of relative position in the original null surface coming from below (say) converts to a spread of arrival times in the orthogonal direction (see Figure 2). The uncertainty of the relative event positions changes depending on a measurement choice of null surface orientation.

Let θ denote the angle of incidence relative to normal. Suppose that every sampling time t_* in the incoming wave an independent error adds holographic uncertainty in the orthogonal direction given by $\Delta t_i^2 = t_* t_P$, as discussed above (Eq. 5). On the reflecting surface, the separation between these events is $t_*/\sin\theta$. The number of terms N in the summed phase offset between measured wavefronts is the number of independent events between them. Between two wavefronts separated by time t we then have $N = t\sin\theta/t_*$ offsets added in quadrature. The standard deviation of position indeterminacy from
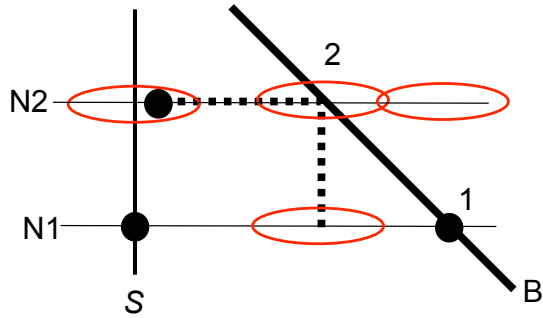
FIG. 2: Holographic encoding of two reflection events on the beamsplitter B. A null surface N is shown at slice N1 corresponding to reflection event 1 and slice N2 corresponding to event 2. Once the position of event 1 is fixed, the relative position of a reflection event 2 on a different slice of the same null surface (the same wavefront in 3D) is encoded by a patch of fields with a finite transverse size given by holographic uncertainty, indicated by an ellipse. The reflection rotates the wavefront to land on a detector surface S where measurement collapses the uncertainty of event location in that direction to a definite value. Again fixed to a reference time from event 1, the arrival time from event 2 inherits the holographic uncertainty in transverse position, observed as a real difference in arrival times from the two events.

each independent event orthogonal to the incoming wave is Δt_i; when projected into arrival time in the direction of the outgoing reflected wave, the standard deviation in arrival time from each one is $\Delta t_i \sin 2\theta$. The wavefront arrival time after time t thus differs by a gaussian random offset from the previous wavefront, with a standard deviation

$$\Delta t = \sqrt{t_P t}\sqrt{\sin \theta} \sin 2\theta. \qquad (7)$$

The actual sampling time t_* cancels out in this result. The interpretation of Eq.(7) is that a signal from beam interference wanders randomly: statistical fluctuations mimic the effect of a random walk of the beamsplitter surface of about a Planck length per Planck time, times some geometrical factors. Note that in this model and in these units, the noise in the signal depends only on the geometric layout of the optical elements, and not on any other parameters such as power, arm length or wavelength.

The geometrical factors in Eq.(7) show how the effect varies for different kinds of reflections. The noise does not appear in normal-incidence optical cavities of an interferometer, and is highly suppressed in near-normal or grazing-incidence reflection. In these cases the transverse position is still uncertain, but it is not measured in the arrival time signal. The geometrical factors in (7) affect the absolute normalization of holographic noise and quantitative tests of holographic theories with interferometer data: the full holographic indeterminacy is not detected, but is always suppressed by a numerical factor depending on the setup.

C. Lower Bound on Holographic Noise from Upper Bound on Gravitational Entropy

The holographic hypothesis is that the world can be described by a quantum theory of energy and spacetime on 2D null surfaces with one quantum degree of freedom per $4l_P^2$, where $t_P \equiv l_P/c \equiv \sqrt{\hbar G_N/c^5}$. We refer to the literature for reviews[11, 13] of the circumstantial theoretical evidence for this idea from black hole thermodynamics and other arguments.

The normalization of Eq.(7) has been chosen to provide a lower bound on noise predicted by theories where the effective quantized states of the system in 2D that specify the 3D geometry take the form of waves or fields in the conventional sense, with a UV cutoff that limits the number of degrees of freedom to that required to satisfy holographic entropy bounds. If covariant entropy bounds are explained in terms of a limited number density of degrees of freedom in a theory built on light sheets[11], arguments based on black hole physics suggest that the number of degrees of freedom must be less than one per $4l_P^2$, or one per $2l_P$ in each dimension. For a theory with a minimum cutoff wavelength λ_{min}, the Nyquist-Shannon sampling criterion implies that any state is specified by its value at two points in position space per λ_{min}. The normalization of the noise adopted in Eq.(7) assumes a maximum of one independent transverse position measurement per l_P in each dimension on the light sheet, with the sampling interval set to the Planck time $t = t_P$. That is sufficient to specify states in a theory based on waves with a cutoff at $\lambda_{min} = 2l_P$: it is thus the smallest noise compatible with having one degree of freedom per $\lambda_{min} = 2l_P$ in each transverse direction, in a theory where a wave mode corresponds to a quantized degree of freedom.

An experimental upper bound on holographic noise gives a lower bound on the number density of independent position eigenstates on a null surface, therefore on the number of degrees of freedom. The noise in Eq.(7) can therefore be regarded as a prediction of the hypothesis that the holographic behavior of General Relativity is explained in terms of the quantum behavior of fields on 2+1D light sheets, where the covariant entropy bound arises from a minimum wavelength.

IV. PREDICTED HOLOGRAPHIC NOISE IN INTERFEROMETER SIGNALS

A. Michelson Interferometer

The currently operating interferometer GEO600 continuously measures the difference of proof mass positions in two orthogonal directions from a beamsplitter, where all components are suspended and effectively in free fall

in the measured directions[19, 22]. The holographic uncertainty is predicted to appear as a noise in the phase signal stream resembling the effect of a classical random walk of the beamsplitter position, along its inclined axis mixing the two orthogonal directions.

The accumulated phase difference between the AB and BC arms of a simple Michelson interferometer is the same as it would be if there were a classical random perturbation of the difference in arm lengths, $\Delta l = AB - BC$, at time t, with variance

$$\langle \Delta l^2 \rangle_H > ctl_P \sin \theta (\sin 2\theta)^2. \tag{8}$$

The zero-parameter prediction (Eq. 8) for holographic noise in terms of equivalent beamsplitter position noise, in absolute physical units, is the main result of this paper. It provides a precise experimental target for direct test of the holographic hypothesis for Planck scale quantum geometry represented by Eq.(3). Should holographic noise exist, its universal spectrum, and the specific dependence on transverse measurement characteristic of its origin, allow diagnostic signatures that distinguish and separate it from other sources of system noise.

B. Equivalent Metric Strain Noise Spectrum for Signal-Recycled Michelson Interferometer

The prediction (Eq. 8) for extra noise in the orthogonal path difference can be translated into an equivalent gravitational-wave strain spectrum to compare with conventional units for quoted noise. A precise calculation requires a transfer function derived from a detailed model of signal formation in the apparatus. We offer a rough estimate here for a simplified model of the GEO600 signal-recycled Michelson interferometer to show that the prediction is a realistic experimental target.

Consider first a frequency $f = c/L$, where L denotes the interferometer arm length. We characterize the holographic fluctuations by h_H^2, the equivalent gravitational-wave metric strain perturbation power spectral density per bandwidth— the gravitational wave spectrum that would give rise to the same signal fluctuations. Consider the passage of a gaussian gravitational wavepacket of frequency $\approx f$ and duration $t \approx 1/f$, normal to the interferometer plane. By the standard definition of h, the passage of this gravitational wavepacket over about one wave cycle causes a variation of fractional path difference between arms related to the broadband spectral density h^2 by

$$\langle \Delta l^2 \rangle_{GW}/l^2|_{f=1/t} \approx h^2 f, \tag{9}$$

where $\langle \Delta l^2 \rangle_{GW}$ is the variance in path difference. Equating this with the holographic uncertainty in beamsplitter position (Eq. 8) for $t = L/c$, we find that the power spectral density per bandwidth of holographic noise at frequency $1/L$ is given by the Planck time, $h_H^2 \approx t_P$.

For lower frequencies, corresponding to many reflections of light in the GEO cavity, the phase signal in GEO approximately represents the average arm length difference measured over time $\approx f^{-1}$. Light arriving at the detector at a given time combines the phase offsets from many traversals of the arms. The position of the beamsplitter at a given instant contributes to— is measured by— this instantaneous phase signal not just once, but many ($\approx c/fL$) times, and it has the same value each time. Holographic uncertainty applies only to one measurement of given event position. In the combined signal, the effect of averaging over many reflections is thus to reduce the total phase offset instead of adding offset in quadrature. Instead of displaying quantum indeterminacy corresponding to the time $\approx f^{-1}$, the difference in phase offset between two times separated by less than a typical photon residence time tends to converge to a mean "true" classical value of the arm difference. The arm length difference standard deviation at frequency $f = 1/t$, with averaging time t, is then $\Delta l \approx (Lct_P)^{1/2}(L/ct)^{1/2}$. (The measured arm length difference for frequency f is a fraction fL/c of what it would be if the arm length were c/f, the same as for a gravitational wave of the same frequency.) Recalling that the equivalent gravitational wave power spectral density h gives $(\Delta l/l)^2 \approx h^2 f$, we find that the equivalent spectrum remains flat, $h_H^2 \approx t_P$, independent of f.

At still lower frequencies, below the inverse residence or averaging time $t_{res} = 1/f_{res}$ contributing to the location of a fringe, the effective spectrum changes slope. Phase offsets at widely separated times are independent measurements. Holographically generated phase error accumulates like a random walk with time, leading to $\Delta l \propto f^{-1/2}$ and $h_H \propto 1/f$. The overall holographic noise spectrum is thus given by

$$h_H \approx \sqrt{t_P} \tag{10}$$

for $f > f_{res}$ and

$$h_H \approx (f_{res}/f)\sqrt{t_P} \tag{11}$$

for $f < f_{res}$. For GEO600, f_{res} can be estimated from the bandwidth of the signal recycling cavity, about 700 Hz. This rough estimate omits numerical factors of the order unity for both f_{res} and overall amplitude, but contains no truly free parameters. Since it predicts noise comparable in magnitude and spectrum to the level of noise measured in the current GEO600 system from about 100 to 600 Hz[30], we conclude that this machine is likely to be capable of either ruling out holographic complementarity (Eq. 3) as a model for quantum geometry, or of studying quantum geometrical fluctuations in detail. The approximate agreement of predicted holographic noise with otherwise unexplained noise in GEO600 motivates further study.

C. Other Interferometer Designs

In other interferometer designs the lengths of two arms are measured separately but are not continuously com-

pared using a beamsplitter. The difference of independently measured arm lengths is sensitive to the classical metric distortion caused by gravitational waves but not to quantum holographic noise. Holographic noise is only introduced when orthogonal positions are compared; except for the occasions where reflection off a beamsplitter contributes a transverse position uncertainty to phase, holographic noise is not added to the phase signal.

For example, the current LIGO design[20, 21] tends to suppress holographic noise relative to gravitational wave signals. LIGO's power is much greater in its two separate Fabry-Perot arm cavities than at the beamsplitter, whereas GEO600 sends the full interferometer power through the beamsplitter. In LIGO, the gravitational wave signal, but not the holographic noise, is magnified by the many normal-incidence bounces the light makes in the cavities. For this reason, even though LIGO's noise level for gravitational wave detection is already well below an equivalent metric strain noise $\sqrt{t_P}$, it does not rule out universal holographic noise at its current sensitivity.

There should similarly be no observable holographic noise in the baseline design for the proposed space mission LISA in Michelson (two arm) mode, since it compares phase signals from separate cavities, each of which has only normal incidence reflections. Further analysis is required to estimate the effect for LISA in three-arm Sagnac mode, which compares phases from travel in two directions around a closed triangle. In any case the holographic noise at LISA frequencies is likely to be signifi-cantly below the confusion noise generated by real cosmic gravitational waves.

If the effect is detected, a new interferometer could be designed specifically to measure the properties of holographic noise. Unlike gravitational waves, measurement of holographic noise does not require a long baseline interferometer. A system built for the purpose of studying holographic noise can have shorter arms than gravitational wave detectors, and therefore use a smaller vacuum system. Holographic noise can be studied above a kilohertz, where natural sources of classical gravitational waves become extremely weak and there has been little motivation to develop gravitational wave detectors. However, high frequency studies will be limited by photon shot noise, and require laser power even larger than current interferometers.

[1] J. Ellis, J. S. Hagelin, D. V. Nanopoulos, and M. Srednicki, Nucl. Phys. B **241**, 381 (1985)
[2] G. Amelino-Camelia, Nature **398**, 216 (1999)
[3] G. Amelino-Camelia, Phys. Rev. D **62**, 024015 (2000)
[4] G. Amelino-Camelia, Nature **410**, 1065 (2001)
[5] G. Amelino-Camelia and C. Lammerzahl, Class. Quant. Grav. **21**, 899 (2004)
[6] G. Amelino-Camelia, C. Lammerzahl, A. Macias and H. Muller, AIP Conf. Proc. **758**, 30 (2005)
[7] L. Smolin, arXiv:hep-th/0605052.
[8] G. 't Hooft, "Dimensional reduction in quantum gravity," arXiv:gr-qc/9310026.
[9] L. Susskind, J. Math. Phys. **36**, 6377 (1995)
[10] G. 't Hooft, "The Holographic Principle," arXiv:hep-th/0003004.
[11] R. Bousso, Rev. Mod. Phys. **74**, 825 (2002)
[12] T. Padmanabhan, Int. J. Mod. Phys. D **15**, 1659 (2006)
[13] T. Padmanabhan and A. Paranjape, Phys. Rev. D **75**, 064004 (2007)
[14] C. J. Hogan, "Quantum Gravitational Uncertainty of Transverse Position," arXiv:astro-ph/0703775.
[15] C. J. Hogan, "Spacetime Indeterminacy and Holographic Noise," arXiv:0706.1999 [gr-qc].
[16] C. J. Hogan, "Holographic Indeterminacy, Uncertainty and Noise," arXiv:0709.0611 [astro-ph].
[17] C. J. Hogan, "Quantum Indeterminacy of Emergent Spacetime," arXiv:0710.4153 [gr-qc].
[18] S. Schiller, C. Lämmerzahl, H. Müller, C. Braxmaier, S. Herrmann, and A. Peters, Phys. Rev. D 69:027504, 2004
[19] J. Hough and S. Rowan, Living Rev. Relativity 3, (2000), 3. URL http://www.livingreviews.org/lrr-2000-3
[20] B. Abbott *et al.* [LIGO Scientific Collaboration], Nucl. Instrum. Meth. A **517**, 154 (2004)
[21] B. Abbott *et al.* [LIGO Scientific Collaboration], Phys. Rev. D **76**, 082003 (2007)
[22] B. Wilke et al., Class. Quantum Grav. 19 (2002), 1377
[23] W. H. Zurek, Phys. Rev. A **76**, 052110 (2007)
[24] E. Nelson, Phys. Rev. **150**, 1079 (1966)
[25] E. P. Wigner, Rev. Mod. Phys. **29**, 255 (1957)
[26] H. Salecker & E. P. Wigner, Phys. Rev. **109**, 571 (1958)
[27] T. Padmanabhan, Class. Quantum Grav. **4**, L107 (1987)
[28] M. Srednicki, Phys. Rev. Lett. **71**, 666 (1993)
[29] A. Yarom and R. Brustein, Nucl. Phys. B **709**, 391 (2005)
[30] K. Strain, "The Status of GEO600", http://www.ligo.caltech.edu/docs/G/G070506-00.pdf